Mathematicians

helping

Art Historians

and

Art Conservators
Three examples:

Frescoes in Eremitani church in Padua, Italy
Massimo Fornasier et al.
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Frescoes in Eremitani church in Padua, Italy
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New (re)attributions based on threadcounts
Rick Johnson et al.
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Virtually aging/rejuvenating paintings
1. Reconstructing destroyed frescoes, Eremitani church in Padua, Italy

Massimo Fornasier et al.
August 1992: start of concerted large-scale effort to reconstruct the frescoes
First: cleaning, stabilizing, cataloguing of all fragments

Next (1995-97): digitization of numbered fragments on 38 CD-ROM
Fragments: number = 80,735

total surface = 77.47 sq. m

typical size: 5 to 6 sq. cm

Original size of lost painted area: > 800 sq. m

Very few contiguous fragments

For each fragment: location unknown

rotation unknown
Fast method needed to place each fragment!

Digitized images:

pixels $\rightarrow$ grey value: number between 0 and 255
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Represent the information in each fragment so that rotation is easy to compute/recognize:

use **circular harmonics**
The circular harmonics make it easy to find the representation of a rotated image $I$:

$I$ can be decomposed into circular harmonics:

$$I = a_1 H_1 + a_2 H_2 + a_3 H_3 + a_4 H_4 + \ldots.$$ 

Then the rotated version $\text{Rot}(I)$ has the form:

$$\text{Rot}(I) = r_1 a_1 H_1 + r_2 a_2 H_2 + r_3 a_3 H_3 + r_4 a_4 H_4 + \ldots.$$
Mantegna Project 2001-04

> 50 volunteers from the universities of Padua, Venezia and Udine, from the departments of Cultural Heritage Conservation, Art History, Literature, Philosophy, Psychology, Political Science, Mathematics and Engineering.
Mantegna Project 2001-04
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Mantegna Project 2001-04
2. Automatic threadcounting for paintings on canvas

Rick Johnson et al.
3. Virtual aging/rejuvenating of paintings on panel: the Ghissi project

Collaboration Duke University & North Carolina Museum of Art
First a digression on how

“Mathematics and Image Analysis for Art Conservation and Art History”

became part of what I do.
About 9 years ago: started work on applications of image analysis to art history
distinguishing style
distinguishing originals from copies
Determine information at different scales
Determine information at different scales

blur
Determine information at different scales and find the difference.
Determine information at different Scales

Difference = A - B
Determine information at different Scales

Difference = A - B
Determine information at different Scales
Based on wavelet features: defined distance between patches of paintings
  → visualize similarity
(movie)
Next: other types of challenges

Joris Dik

and

Koen Janssen
Van Gogh: Patch of Grass (Paris period)
Van Gogh: 
Patch of Grass 
(Paris period)

X-ray shows portrait from Nuenen period
Reunited

An art historical and digital adventure
Ghissi altarpiece: 3 panels in NCMA
1 in Portland Art Museum
3 in Metropolitan Museum of Art, NY
1 in Art Institute, Chicago
1 missing ...
Charlotte Caspers
Artist
Art Conservator
Art Reconstruction
commissioned by NCMA to recreate the missing panel
The story followed the life of St. John the Evangelist as described in the “Golden Legend”, a medieval bestseller.

This allowed a guess for the topic of the missing scene
New panel is gorgeous – fresh colors, gleaming gold and punchmarks.

Also: documentary made of its realization.
Problem: new panel is so beautiful: vivid colors, shiny gold, sparkling accents

Authentic 14\textsuperscript{th} century panels would look dull and faded in comparison …

Use image analysis and image processing to “age” the panel; aged copy can then be displayed next to old panels, and new panel separately.
To do “virtual aging”:

* determine color correspondences, and use them to “remap” colors
* introduce cracks (need to study existing craquelure for this)
* “age” gold-leaf work

Printout of virtually aged Caspers panel was used for the “Reunited” exhibition in NCMA, Sept 2016-March 2017, and now in Portland Museum of Art (Oregon)
Can then also “virtually rejuvenate” old panels:

* detect and inpaint cracks
* using color correspondences determined earlier, “remap” old colors to “new”
* rejuvenate gold-leaf work
Studying and Removing Cracks

Craquelure: very dense – not easy to detect automatically all cracks and only cracks …
Previous experience in doing this

Ghent Altarpiece

15th century, Flemish

Jan Van Eyck
Previous experience in doing this
Previous experience in doing this
Previous experience in doing this
Color Remapping
Color Remapping
Color Remapping

Old colors

Rejuvenated colors
Color Remapping

Aged colors

New, fresh colors
Crack Generation
Crack Generation
Putting “new” gilding on rejuvenated panels
Computer graphics techniques were used to model and render the reflections of the gold background and off the punchmarks, once their locations were identified.
(show movie)